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TREE-RINGS AND SUNSPOT NUMBERS

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ABSTRACT

Tree-ring series that record climatic variation have long been of interest for study of possible effects of solar variability on terrestrial phenomena. Spectral analysis, harmonic dial analysis, digital filtering, cross-correlation and principal component analysis were used separately and in combination in an attempt to detect relationships between the annual Wolf sunspot numbers and ring-width indices, primarily from western North America. The results show no evidence of significant, consistent relationships between tree-ring data and sunspot numbers.

INTRODUCTION

This report gives the results of a study of possible relationships between sunspot numbers and tree-ring indices. It was undertaken because longer series of annual sunspot numbers were needed to test certain models for prediction of future solar activity. We felt that tree-ring and sunspot data might be sufficiently correlated to permit the estimation of sunspot numbers from tree-ring data for a long period prior to the beginning of the oldest continuous sunspot records.

The search for periodicities or trends in tree-ring series which might be related to the behavior of the sun is based on the hypothesis that solar variation causes variations in climate, which in turn affect the growth of trees (Douglass 1919, 1928, 1936). It has been amply demonstrated that the variation in widths of annual rings of certain trees from environmentally limiting sites contain long records of climatic fluctuations. In southwestern North America a number of dry-site trees (those which are frequently limited by drought) are usually selected from within a particular area. Their rings are dated and the ring widths are measured and standardized. The data are then pooled in such a way as to maximize the climatic "signal" and minimize the non-climatic "noise" (Fritts 1969). If such data from such a stratified sample of trees are properly treated (Fritts 1969), the resultant tree-ring chronology of wide and narrow growth rings is highly related to climate (Fritts 1965; Julian and Fritts 1968).

There is a long history of the tree-ring and sunspot studies. The possibility of using long records of tree growth to extend sunspot records intrigued the astronomer A. E. Douglass (1919, 1928, 1936). He devoted a major part of his life to the development of the science of dendrochronology and to the search for cycles in tree-ring data. Although a voluminous literature has grown up around this topic since the turn of the century, the evidence for a relationship has not been conclusive, and many have remained skeptical.

In this investigation, several different methods of time series analysis were applied to test for possible associations between variations in sunspot numbers and fluctuations in tree growth. Power spectrum and cross-spectrum analyses were used to search for possible short period cycles or oscillations in tree-ring data which might be related to the well known 11-year sunspot rhythm, or to the 22-year "Hale cycle." The harmonic dial was used in conjunction with a series of digital filtering functions to test for both short and long period cyclic behavior in tree-ring series. The possibility of associations between the very long period or non-periodic variation in sunspot numbers and in that the tree-ring data were further tested by simple linear cross-correlation of the smoothed records.

The various tests were applied not only to tree-ring chronologies for individual sites, but also to the amplitude series derived by principal component analysis of chronologies from throughout western North America (LaMarche and Fritts 1971). The most important series expresses the large-scale changes in ring widths referred to as the characteristic tree-growth anomaly patterns. They are less subject to error than single site chronologies and represent the variation of climatic patterns on a regional scale.

Most of the tree-ring chronologies used in the early phases of this investigation were selected from a set of 26 chronologies from Canada, the United States, and Mexico which were previously used in a study of climatic changes in western North America (Fritts 1965). However, some of these chronologies ended as early as 1930, some combined data from more than one species, and others were based on comparatively small samples. While this particular investigation was under way, there was a major effort by the Laboratory of Tree-Ring Research to increase the number and improve the quality of representative North American tree-ring chronologies, as well as bring them more nearly up to date. As these chronologies became available, they were incorporated into a new data set, making a total of 49 chronologies (Stokes and others 1973), which provided the basis for certain later analyses. In a few cases, other chronologies were used for particular tests. The published or unpublished sources are cited where these results are given.

The sunspot data used were the annual values of the Zurich relative sunspot numbers (Wolf sunspot numbers) as given by Waldemier (1961) for the period A.D. 1700-1960. This record is thought to be highly reliable back to about 1830, less reliable back to 1749, and rather crude to 1700 (Allen 1964).

SUMMARY OF TECHNIQUES

Power Spectrum Analysis

A high proportion of the variance in sunspot numbers is associated with a quasi-periodic rhythm having an average frequency of about 1/11 cycles per year (cpy) (Figure 1). Power spectrum analysis is designed for the study of such rhythmic behavior in a time series. To obtain a power spectrum of an annual series, the autocorrelation coefficient is computed for a lag of one year, two years, three years, and so forth up to a predetermined maximum number of lags. Harmonic analysis of the autocorrelation function yields estimates of the relative spectral density at each of a number of frequencies. These "raw" spectral estimates are then smoothed using a moving-average filter. Several types of filter weights are in common use (Jenkins and Watts 1968). The resulting smoothed spectrum will contain sharp peaks at frequencies corresponding to exact periodicities in the original series, and broad humps where the variation in the series is less regular. In addition, the power spectrum of a time series containing persistence will show noticeable trend upon which the spectral peaks are superimposed.

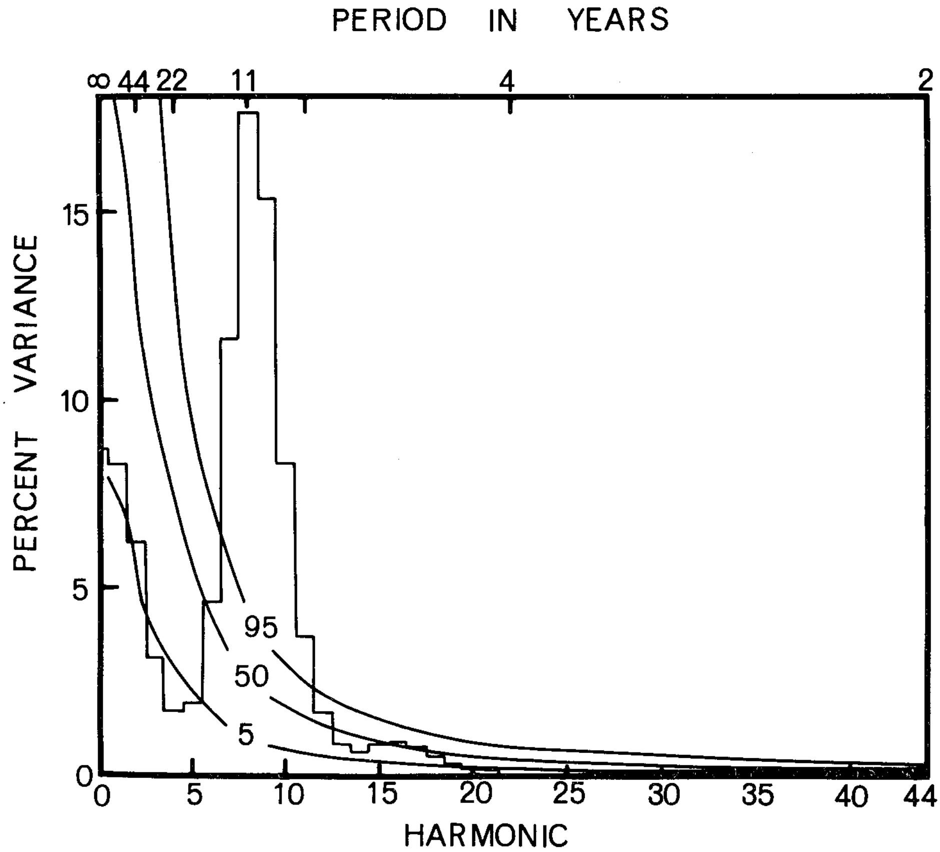

Figure 1. Power spectrum of annual sunspot numbers, based on 44-lag analysis using Parzen weights. Smooth lines give 5 percent, 50 percent, and 90 percent confidence limits for spectral estimates from a Markov process having a first-order autocorrelation coefficient equal to that observed for the time series (Mitchell and others 1966).

The total number and particular frequencies at which spectral estimates are obtained is determined by the maximum number of lags for which the autocorrelation coefficient is computed. In general, increasing the maximum number of lags improves the spectral resolution, but also increases the possibility of obtaining a high spectral density at some frequency by chance alone. In this study, most analyses were carried out to 44 lags, but in no case did the number of lags exceed 1/4 the number of observations in the series. Using a number of lags equal to a multiple of 11 years ensured that one spectral estimate would be centered in the frequency range of particular interest, in this case that of the 11-year sunspot rhythm.

A peak in the power spectrum of a time series cannot be accepted as proof of an underlying periodicity unless it can be shown that this result is unlikely to have been obtained by chance alone. A procedure for testing the statistical significance of peaks in power spectra has been outlined by Mitchell and others (1966) and was followed in this work. It is known that tree-ring series normally exhibit relatively high first-order autocorrelation (Matalas 1962). The effect of this autocorrelation is to inflate the spectral densities in the low-frequency range. Using the observed first-order autocorrelation coefficient for the series, a null continuum is calculated which represents the theoretical power spectrum for a series containing only first-order Markov persistence in addition to random variation. Statistically significant peaks in the power spectrum are those lying outside the confidence limits which parallel the calculated null continuum (see Figure 1).

Digital Filtering

Filtering and smoothing functions are used in order to emphasize variation in a time series within particular frequency ranges. The justification for filtering is the assumption that variation at other frequencies is either random error, or is of no significance to the particular type of evaluation being carried out. The filtered value of an observation in a time series is thus an estimate of what the value would have been if variation at the other frequencies had not been present. Recent developments, made possible by high speed computers, have eliminated many of the objectionable features associated with simple moving averages. Filters can now be designed which will preserve the phase and amplitude characteristics of the original series, within any desired frequency range.

The filtered value of any observation x_t is calculated according to the equation

$$\hat{x}_t = \sum_{k=-n}^{m} W_k X_{t+k}$$

$$= W_{-n} X_{t-n} + \dots + W_o X_t + \dots W_m X_{t-m}$$

where W_k is a particular weight in the smoothing function, and W_o is termed the central or principal weight. An important characteristic of any filtering function is its frequency response curve, defined by the ratio of the amplitude of a sine wave before filtering to its amplitude after filtering, at each frequency. Techniques for the selection of filter weights giving a desired frequency response are presented by Holloway (1958).

Four digital filters were used in this study. The filter weights are listed in Table 1 and their frequency response curves are shown in Figure 2. The "high-pass" filter removes most of the variation at frequencies lower than 1/8 cpy. The "low-A" filter is the reciprocal of the high pass filter, and removes most of the variation at frequencies higher than 1/8 cpy. The "11-year band-pass" filter, as given by Brier (1961), retains variation with periods of 6 to 15 years and suppresses variation outside this range. The "low-B" filter eliminates most of the short-period oscillations while retaining long-term periodicities and trends. The effects of filtering are illustrated in Figure 3, which shows the Wolf sunspot numbers from 1700 to 1960 plotted in their original form, and after filtering with the 11-year band pass filter and with the low-B filter, respectively.

Harmonic Dial

One type of analysis that utilizes time series which have been smoothed by a digital filtering operation is the construction of a harmonic dial. Prefiltering of the two series to be compared emphasizes oscillatory movements within the frequency range of interest, without modifying the original phase relationships. To construct a harmonic dial, the dates of maxima in one series are converted into indices representing their occurrence

Table 1. Weights for four digital filters.

M	HIGH-PASS	LOW-A	11-YEAR BAND PASS	LOW-B
0	.7744	.2256	.1360	.0798
±1	−.1933	.1933	.1070	.0782
2	−.1208	.1208	.0340	.0737
3	−.0537	.0537	−.0460	.0667
4	−.0161	.0161	−.0950	.0581
5	−.0030	.0030	−.0980	.0486
6	−.0003	.0003	−.0620	.0390
7			−.0190	.0301
8			−.0230	.0223
9			.0380	.0159
10			.0340	.0109
11			.0230	.0071
12			.0130	.0045
13			.0070	.0027
14			.0030	.0016
15			−.0010	.0007
16			−.0060	
17			−.0080	
18			−.0090	
19			−.0060	
20			−.0040	
21			−.0010	

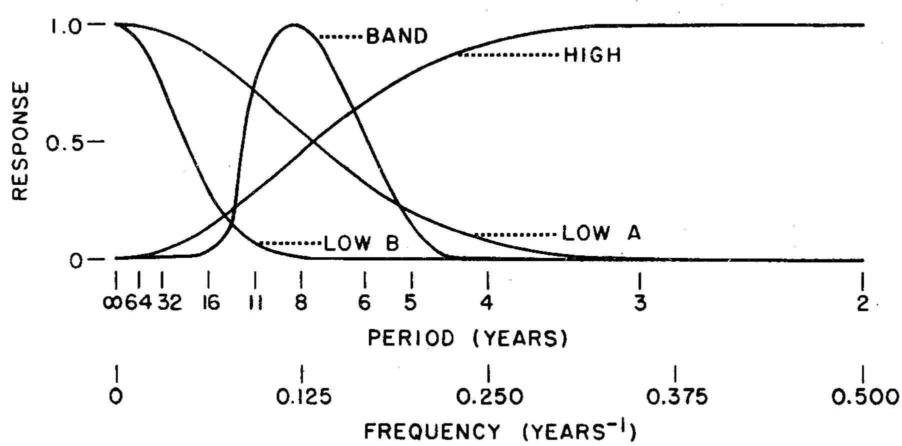

Figure 2. Frequency response functions for four digital filters.

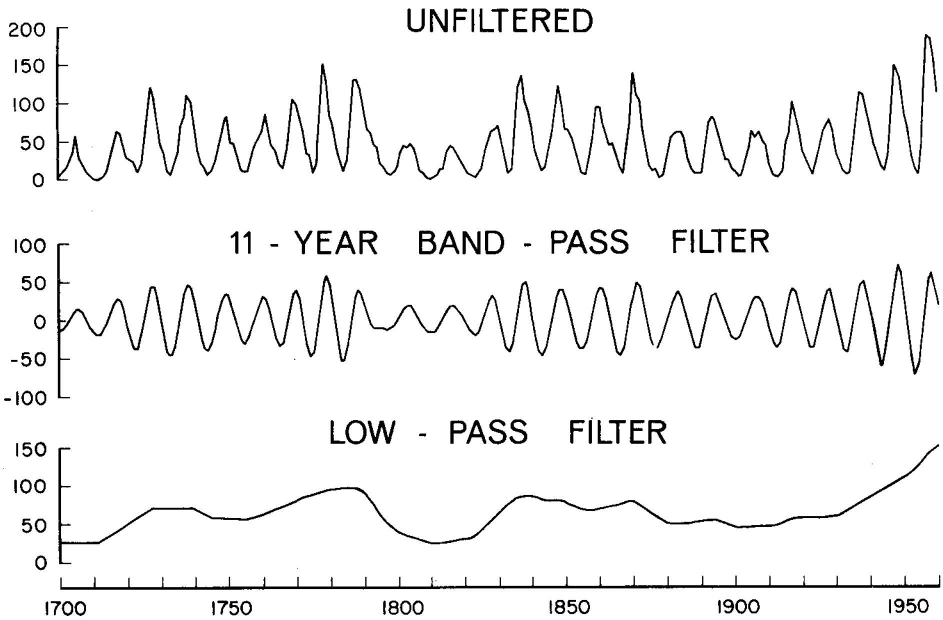

Figure 3. Annual sunspot numbers plotted in original form, and after smoothing with 11-year band-pass and low-B filters, respectively.

relative to peaks in another cyclical series, where $\theta_s = 0$ indicates the beginning of the cycle, $\theta_s = 90°$ indicates 25 percent of the way through the cycle, $\theta_s = 180°$ indicates half way through the cycle, etc. The values of θ_s and the magnitude of the maxima in the first series are then plotted on a circular diagram on which radial distance represents amplitude and angular distance represents phase (Brier 1961).

Correlation and Regression

The simple linear correlation coefficient can be used to test the association between two time series. In this work, individual tree-ring chronologies, as well as amplitude series derived by principal component analysis of tree-ring data, were compared with the series of annual Wolf sunspot numbers. Testing the significance of the correlation coefficients obtained from comparison of these time series is complicated by the fact that each normally contains significant persistence. Although techniques exist which enable one to correct for the effect of persistence, this was not found to be necessary because very few of the correlation coefficients obtained exceeded the limits expected for completely uncorrelated time series.

Correlation was also made after smoothing both the sunspot and tree-ring series, using the two low-pass digital filters described above. In this case, the effective number of observations in each series is greatly reduced, with a corresponding increase in the absolute value of the correlation coefficient required to demonstrate a statistically significant correlation (Rodriguez-Iturbé and Yevdjevich 1968). A conservative estimate of the effective number of observations in the filtered series was obtained according to the formula

$$N' = N/k$$

where N is the number of observations in the original series and k is the number of filter weights used.

Principal Component Analysis

Principal component analysis – also termed the "eigenvector" or "empirical orthogonal function" approach – was used in this investigation to determine whether large scale patterns of tree-growth variation show any demonstrable response to changing levels of solar activity. This method of analysis was introduced into meteorology by Lorenz (1956), and is being increasingly applied to problems in this field (Grimmer 1963; Stidd 1967; Kutzbach 1967; Fritts and others 1971). Basically, it is a method for reducing the number of variables that need to be specified in order to "explain" a major proportion of the total variance in a set of data, by taking advantage of the intercorrelations that exist in the original data field.

The underlying theory and a procedure which closely parallels that used in this study are discussed in detail by Sellers (1968). In our application, the data field consists of a two-dimensional array of observation points (tree-ring collection sites). The observations are the time series of annual tree-ring indices at each point after normalization by subtracting the chronology mean and dividing by the chronology standard deviation. A contour map of the components of each eigenvector resulting from the analysis represents a characteristic tree-growth anomaly pattern (LaMarche and Fritts 1971). The amplitude of the eigenvector, computed from the normalized tree-ring indices, is a function which shows how the relative importance of each characteristic pattern changes through time. An important attribute of these amplitude series is that they are statistically uncorrelated.

RESULTS

Short Period Oscillations

The power spectrum of sunspot numbers (Figure 1) shows that most of the variance is concentrated in a frequency range of 1/15 to 1/8 cpy, with a peak at 1/11 cpy. If there is variation in tree-ring growth associated with this short period solar variation, the power spectra of tree-ring series would be expected to show corresponding peaks, and there would be clustering of points in a certain quadrant of the harmonic dial. In Figures 4 and 5 power spectra of two long tree-ring series illustrate typical results. In both cases, significant spectral peaks occur, corresponding to long-period variation in the series. However, in neither spectrum is there any suggestion of an 11-year oscillation. Also there was no significant clustering in the harmonic dial.

Another solar rhythm, with an average period of about 22 years, has been thought to be reflected in some terrestrial time series. This is the Hale or "double-sunspot" cycle, based on the observation that the magnetic polarity of sunspot groups reverses between

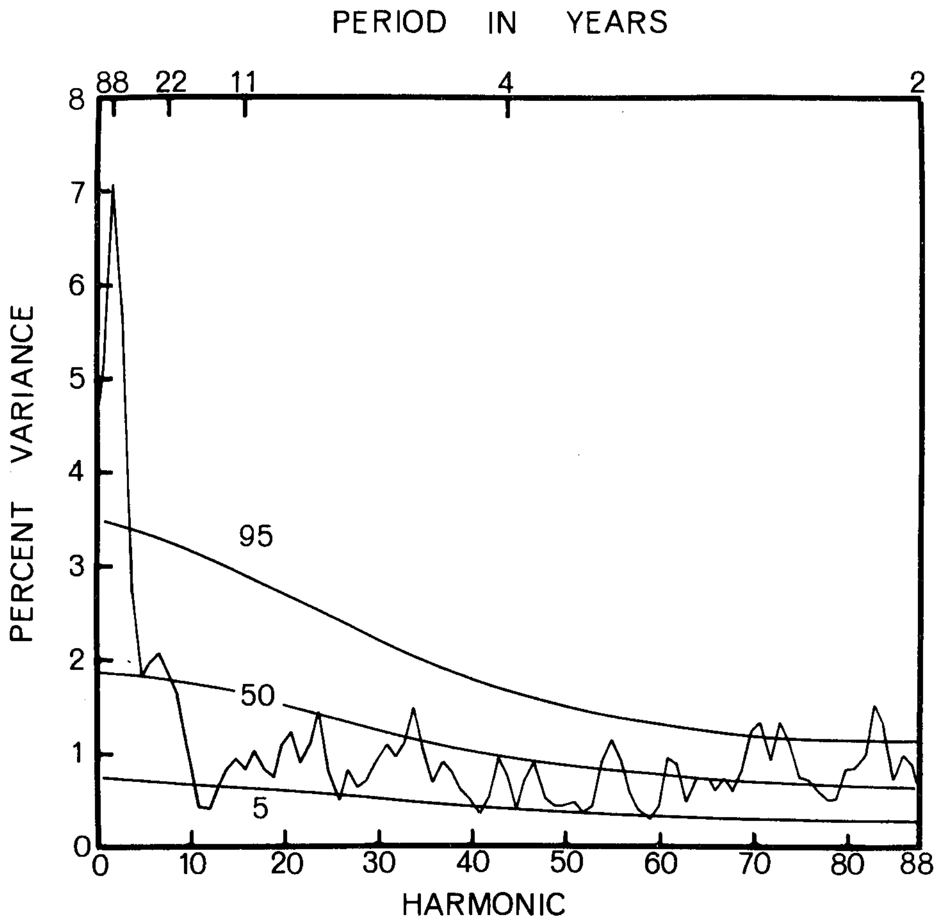

Figure 4. Power spectrum of tree-ring indices from 464 year-old Douglas-fir, Mesa Verde National Park, Colorado. Period is A.D. 1400 to 1963. Analysis carried out to 88 lags with Tukey-Hanning weights.

successive maxima (Bray and Loughead 1967). Neither of the power spectra show significant spectral peaks at this frequency.

Power spectra were also obtained from analysis of shorter tree-ring series. A set of 17 were analyzed – nine from the Rocky Mountain region and eight from areas along the Pacific Coast. The distribution of normalized spectral densities at frequencies of 1/11 cpy and 1/22 cpy is shown in Figure 6. None are significant, as all of the spectral estimates fall within the range expected for series containing only first-order Markov persistence in addition to random "noise."

The harmonic dial was also used to test for relationship between the sunspot rhythm and the variations in tree-growth indices. A number of tree-ring chronologies from western North America, including several published series from the Arctic (Giddings 1947; Sirén 1961), were smoothed using the 11-year band-pass filter (Table 1 and Figure 2). The maxima in the plotted, smoothed series were compared with those of the similarly filtered sunspot series (Figure 3), following the method of Brier (1961). In none

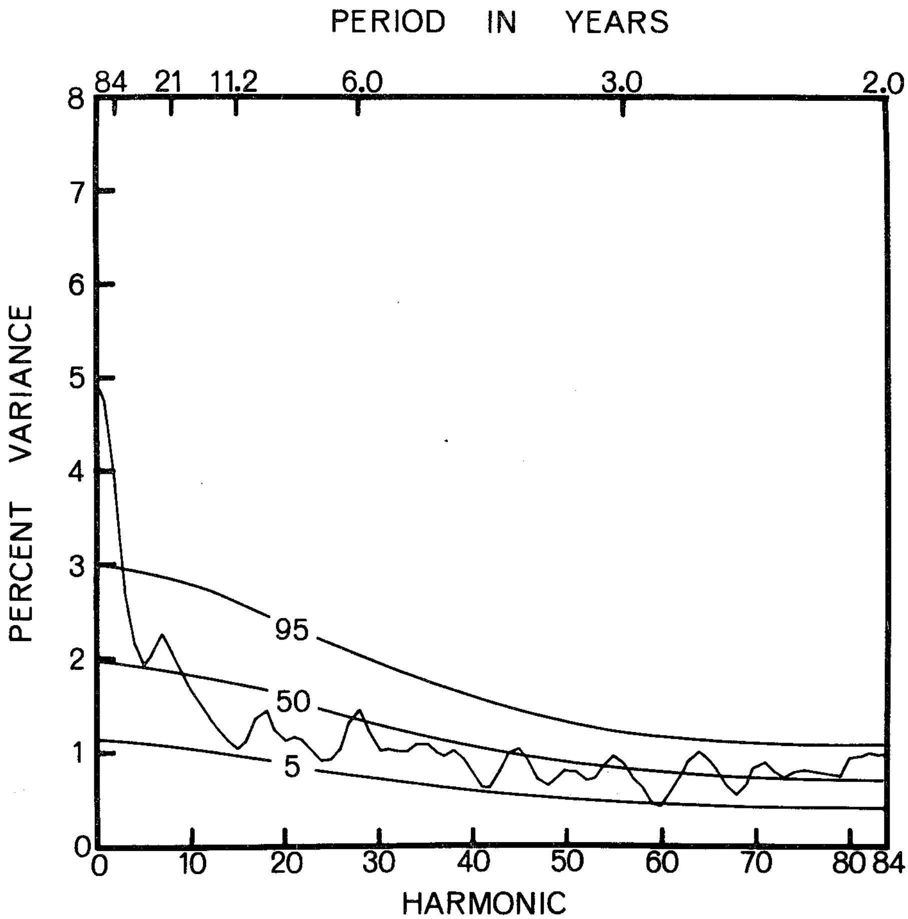

Figure 5. Power spectrum of ring-width index chronology for bristlecone pine, White Mountains, California. Period A.D. 801 to 1954 (Stokes and others 1973). Analysis carried out to 84 lags with Tukey-Hanning weights.

of the analyses was there any significant clustering of points which might indicate a consistent phase relationship between peaks of the 11-year sunspot rhythm and the peaks of the smoothed tree-ring series. The peaks would sometimes coincide for a few cycles but would shift to other phase angles so the long records which give adequate replication turned out to exhibit no consistent relationship. Spectral analysis of the long tree-ring record from northern Sweden (Sirén 1961) also fails to indicate an 11-year periodicity, although spectral peaks at other frequencies could be related to solar variation (Sirén and Hari 1971).

Regional Growth Anomaly Patterns

Power spectrum analysis was used to test the possibility that patterns of tree-growth anomalies on a continent-wide scale might show variations associated with changes in sunspot numbers. For the initial study, 23 of the original set of 26

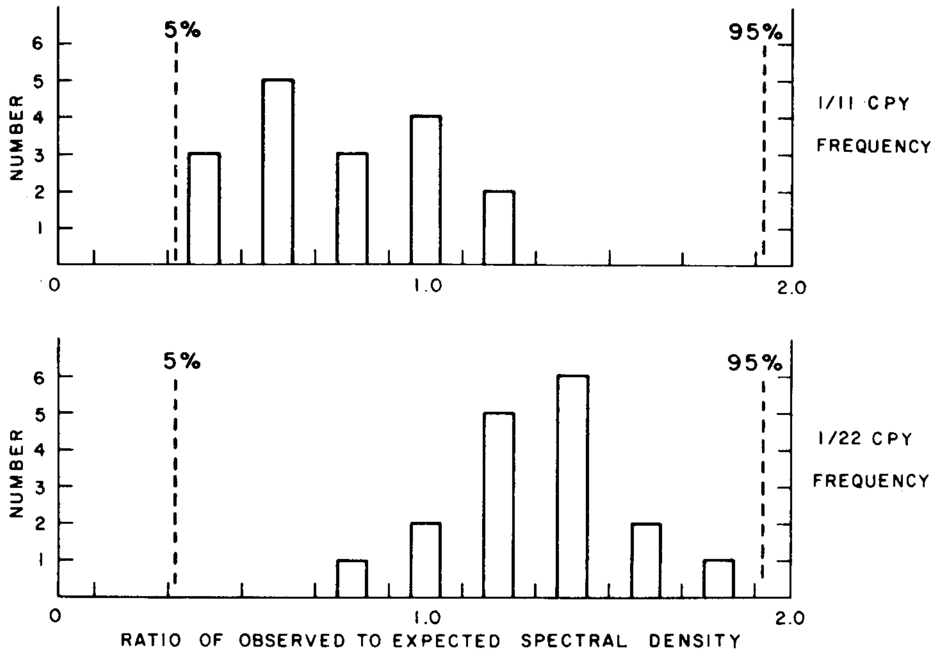

Figure 6. Distribution of spectral estimates at 1/11 cpy and 1/22 cpy for 17 North American tree-ring chronologies expressed as the ratio between observed spectral density and density expected for a Gauss-Markov process having the same first-order autocorrelation coefficient. 5 percent and 95 percent confidence limits calculated according to Mitchell and others 1966.

chronologies were selected for their greater length. Using tree-ring indices for the period 1541-1930, principal component analysis yielded 23 eigenvectors. The seven most important accounted for about 65 percent of the total variance in the original 23 series. Power spectrum analysis of the amplitude functions of these seven eigenvectors, to a maximum of 44 lags, produced results of potential significance to this study. The power spectrum of the amplitudes of the first eigenvector contained a pair of peaks corresponding to frequencies of 22.0 and 29.2 years. Following the procedure outlined by Mitchell and others (1966) the joint occurrence of these two peaks was found to be significant at the 99 percent confidence level, under the hypothesis that variation in this frequency range might be expected *a priori* from knowledge of similar variation in the sunspot series.

Two alternative schemes were used to convert the Zurich yearly mean sunspot numbers into a series reflecting the 22-year magnetic polarity cycle. First, the sunspot numbers were given negative signs in alternate cycles, with the sign of the 1957 maximum considered positive. Then, the method proposed by Jose (1965) was used, in which certain minor maxima are considered to be parts of longer cycles. The series have very similar power spectra, but differ in phase between 1797 and 1876. The amplitude of the first eigenvector is plotted for comparison with the modified sunspot record in Figure 7.

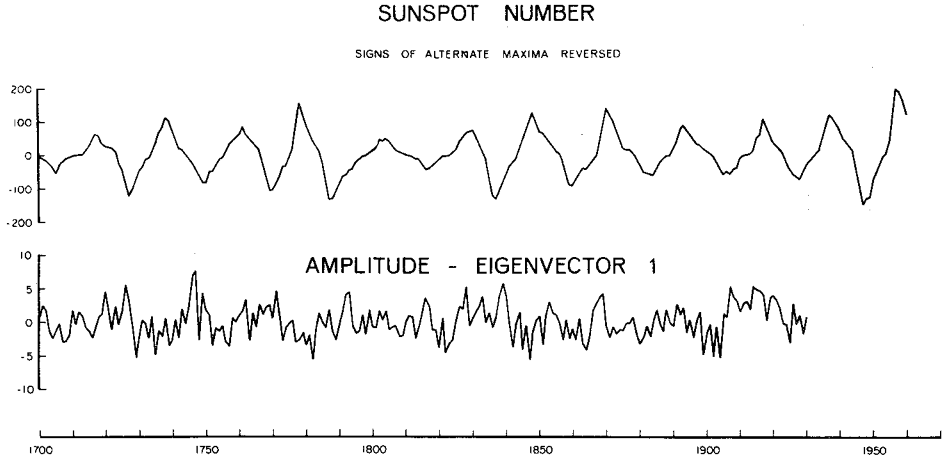

Figure 7. Amplitude of first eigenvector of tree growth, 1700-1930, plotted for comparison with the modified sunspot record (signs of alternate maxima reversed). There is a recognizable oscillation of the amplitude series with a period of 20 to 30 years, but it appears unrelated to the "double sunspot cycle."

To test the possibility that the 20 to 29 year peak in the spectrum of the first amplitude series might be related to the "double sunspot cycle," the coherence (Jenkins and Watts 1968) between the amplitude series and each of the converted sunspot series was computed. There proved to be no significant coherence. Therefore, the 22-29 year oscillation in the first eigenvector amplitude is apparently unrelated to solar variability. It may, however, reflect other extraterrestrial influences. Brier (1968) has presented evidence for a 27-year repeat pattern in the value of the zonal index. He tentatively attributes this to the seasonal modulation of an 11.6-month cycle in the solar-lunar gravitational tide.

Long Period Variation

Much of the variance in sunspot numbers is due to variations over periods much longer than that of the 11-year rhythm. A graph of the yearly mean sunspot numbers (Figure 3) shows that, although the minimum sunspot numbers remain nearly constant from one cycle to the next, the maximum numbers tend to increase or decrease regularly for several consecutive cycles. Removal of the 11-year component through use of a low-pass filter emphasizes this feature.

As shown by their power spectra, many tree-ring series also contain significant low-frequency variance. It is thus conceivable that general trends in the level of solar activity might be reflected in tree-ring growth. The first method that we used to test this possibility was simple correlation after pre-filtering to remove all or part of the 11-year periodicity from the sunspot series, and to remove high-frequency "noise" from the tree-ring series. Both the low-A and low-B filters were used. The distribution of the resulting simple correlation coefficients is shown in Figure 8. In 26 trials using the low-A filter (with a 50 percent response at 8 years) none of the correlation coefficients was

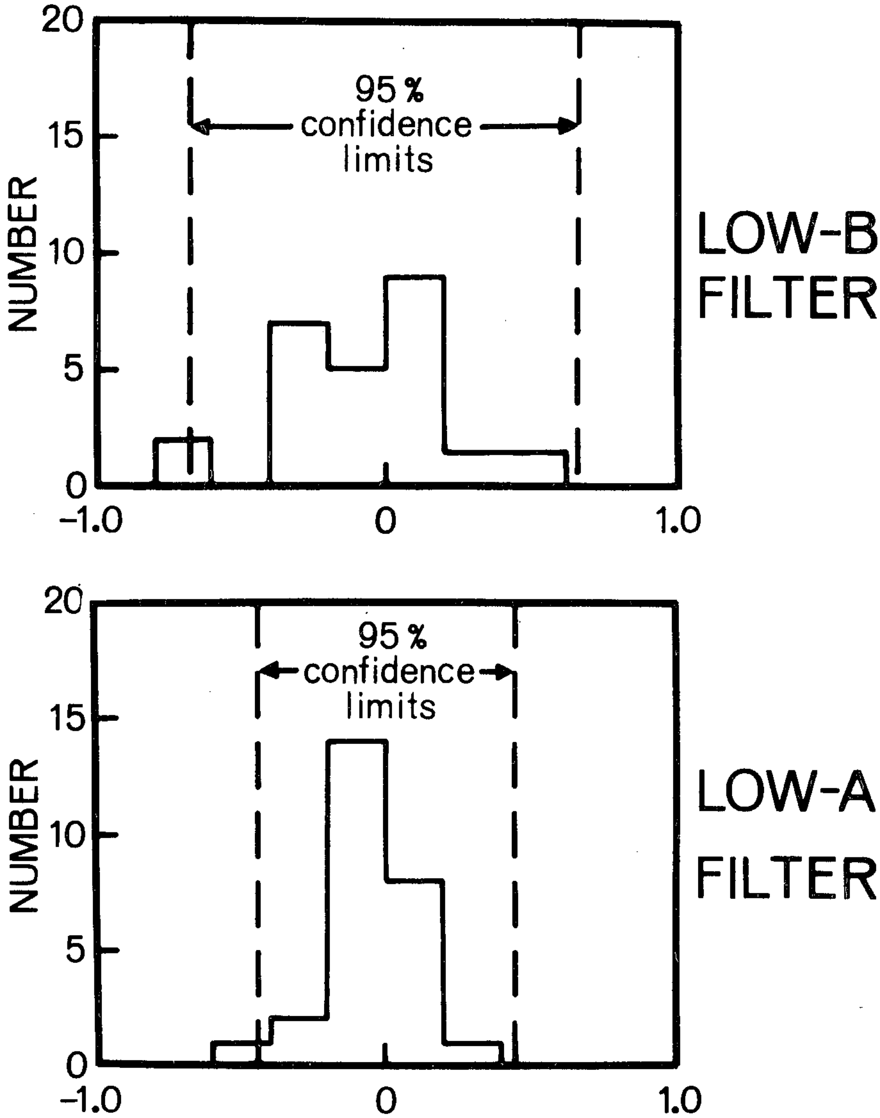

Figure 8. Distribution of correlation coefficients between smoothed ring-width index series and the record of annual sunspot numbers. High-frequency variation was removed from the series using the low-A and low-B filters, respectively, prior to correlation. Confidence limits were calculated using the estimated effective number of observations, N′, described in the text.

found to be significant. Using the low-B filter (with a 50 percent response at 28 years) only one result exceeded the confidence limit, an expectable result in a large number of trials. It must be concluded that there is no evidence from these calculations of an in-phase relationship between individual tree-ring chronologies and the smoothed sunspot record.

The harmonic dial was again used to search for possible out-of-phase relationships of tree growth and long period solar variation. The maxima from five long tree-ring series from bristlecone pine (*Pinus longaeva*), sequoia (*Sequoia gigantea*), limber pine (*Pinus flexilis*), and Douglas-fir (*Pseudotsuga menziesii*), after smoothing with the low-B filter, were converted to indices representing the locations relative to maximum in the 179-year cycle of Jose (1965), where θ_j represents the phasing within each 179-year cycle analogous to the phasing of the 11-year cycle described previously. If there is any significant association of tree-ring maxima with the particular cycle in question, the plots on the harmonic dial will show a clustering of values at a particular quadrant of the dial (see Brier 1961, Figures 9 and 10). However, the plotted tree-ring maxima were scattered randomly around the dial. These results show no evidence for phase relationships between these tree-ring series and a 179-year solar cycle.

CONCLUSIONS

The results of our investigation offer no convincing evidence of consistent relationships between ring width and solar variation. The absence of significant peaks in power spectra or of phasing in the harmonic dial corresponding to the 11-year sunspot rhythm or the 22-year "double cycle" indicate that no persistent relationship with the Zurich sunspot numbers exists in any of the tree-ring series studied. Furthermore, the power spectra of the amplitude series, reflecting variations in major regional growth-anomaly patterns in western North America, show no significant spectral peaks at the 1/11 cpy frequency. However, the results of the power spectrum analyses do not rule out the possibility of solar-related oscillations in tree-ring series which undergo frequent phase shifts or reversals. Neither do these results completely exclude the possibility that longer period variation in the level of solar activity is reflected in tree-growth variation. However, the results of correlation tests comparing smoothed sunspot data with tree-ring series and with eigenvector amplitude series gave essentially negative results.

From the results of this investigation and from the similar results of other studies (Bryson and Dutton 1961; Rodriguez-Iturbé and Yevdjevich 1968), we conclude that a continued search for empirical associations between terrestrial time series, such as tree-ring indices, and the record of sunspot numbers is likely to prove unrewarding. However, long tree-ring records are available or are being developed for many species over a large part of the globe. The expanding data base, the improved understanding of the relationships between climatic variables and tree-ring growth, coupled with more powerful analytical techniques, may permit testing of geophysical hypotheses linking solar variations to climate over time periods longer than those encompassed by climatic or sunspot records.

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